

Confidence Intervals 2019 Fall School

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Disclaimer: It has come to our attention that there was a mathematical error on slides 16, 17 and 23 which should have used division rather than multiplication in the equation.

- Ratio Study
- Median
- Mean
- Coefficient of Dispersion
- Sample Size Adjusted Maximum Coefficient of Dispersion
- Upper and Lower Confidence Intervals
- Conversion into 100% Aggregate Assessed Values

Basic Assessment Statistics

- Ratio Study: IAAO Standard on Ratio Studies, Page 43

“A study of the relationship between appraised or assessed values and market value.

Indicators of market values may be either sales (sales ratio study) or independent

“expert” appraisals (appraisal ratio study).”

Median: Fundamentals of Mass

Appraisal, Page 112

“Midpoint or middle value when data is arrayed in order or magnitude.”

$$(N + 1) / 2 = \text{Median}$$

Definitions

Mean: Fundamentals of Mass Appraisal,
Page 113

“The mean is the average value. It is found by summing up the values of each item in the sample and dividing by the number of items.”

(Sum of X observations) / N = Mean

Definitions

Coefficient of Dispersion: IAAO Standard on Ratio Studies, Page 13

“The COD measures the average percentage deviation of the ratios from the median ratio and is calculated by the following steps:”

$$\text{Sum ABS}(X \text{ observation} - \text{Median}) / N / \text{Median} = \text{COD}$$

IAAO Standard for commercial property 5%-20%

Page 17

Definitions

- Sample Size Adjusted Maximum COD:

Confidence Intervals for the COD
Limitations and Solutions

by Robert J. Gloudemans

See Page 2 of January 28, 2019 memo
from Department of Revenue

Table 1. Tolerance Factors and Maximum Acceptable CODs

| N | 90% Tol Fac | Max COD | 95% Tol Fac | Max COD |
|----------|--------------------|----------------|--------------------|----------------|
| 5 | 1.395 | 27.89 | 1.540 | 30.80 |
| 6 | 1.359 | 27.18 | 1.488 | 29.76 |
| 7 | 1.332 | 26.64 | 1.449 | 28.97 |
| 8 | 1.310 | 26.20 | 1.418 | 28.35 |
| 9 | 1.292 | 25.85 | 1.392 | 27.85 |
| 10 | 1.277 | 25.55 | 1.371 | 27.42 |
| 11 | 1.264 | 25.29 | 1.353 | 27.06 |
| 12 | 1.253 | 25.06 | 1.337 | 26.75 |
| 13 | 1.243 | 24.87 | 1.324 | 26.47 |
| 14 | 1.235 | 24.69 | 1.312 | 26.23 |
| 15 | 1.227 | 24.53 | 1.301 | 26.01 |
| 16 | 1.219 | 24.39 | 1.291 | 25.82 |
| 17 | 1.213 | 24.26 | 1.282 | 25.64 |
| 18 | 1.207 | 24.14 | 1.274 | 25.48 |
| 19 | 1.202 | 24.03 | 1.266 | 25.33 |
| 20 | 1.197 | 23.93 | 1.260 | 25.19 |

- 90% Confidence Intervals:

Fundamentals of Mass Appraisal,
Page 365

$(\text{SQRT } N * 1.645)/2$ for
Even Counts

$(\text{SQRT } N * 1.645)/2 + .5$
Odd Counts



1st Example COD is Compliant

| N | Total Assessed Value | Sale Price | Ratio |
|----|----------------------|------------|--------|
| 1 | 154,950 | 285,000 | 0.5437 |
| 2 | 51,290 | 82,500 | 0.6217 |
| 3 | 172,010 | 261,552 | 0.6577 |
| 4 | 74,690 | 110,634 | 0.6751 |
| 5 | 247,530 | 343,331 | 0.7210 |
| 6 | 29,960 | 40,484 | 0.7400 |
| 7 | 74,620 | 98,000 | 0.7614 |
| 8 | 132,400 | 171,738 | 0.7709 |
| 9 | 153,650 | 192,500 | 0.7982 |
| 10 | 249,910 | 310,000 | 0.8062 |
| 11 | 205,500 | 251,368 | 0.8175 |
| 12 | 52,050 | 62,288 | 0.8356 |
| 13 | 1,838,600 | 1,978,180 | 0.9294 |
| 14 | 224,900 | 235,000 | 0.9570 |
| 15 | 173,470 | 152,691 | 1.1361 |
| 16 | 141,960 | 116,750 | 1.2159 |
| 17 | 69,190 | 47,000 | 1.4721 |

| N | Ratio |
|----|---------------|
| 1 | 0.5437 |
| 2 | 0.6217 |
| 3 | 0.6577 |
| 4 | 0.6751 |
| 5 | 0.7210 |
| 6 | 0.7400 |
| 7 | 0.7614 |
| 8 | 0.7709 |
| 9 | 0.7982 Median |
| 10 | 0.8062 |
| 11 | 0.8175 |
| 12 | 0.8356 |
| 13 | 0.9294 |
| 14 | 0.9570 |
| 15 | 1.1361 |
| 16 | 1.2159 |
| 17 | 1.4721 |

| | |
|--------|--------|
| Median | 0.7982 |
| Mean | 0.8506 |
| N | 17 |

$$\text{Median} = (N + 1) / 2$$

$$(17 + 1) / 2 = 9$$

9th Observation

| N | Ratio | X-Med |
|----|--------|--------|
| 1 | 0.5437 | 0.2545 |
| 2 | 0.6217 | 0.1765 |
| 3 | 0.6577 | 0.1405 |
| 4 | 0.6751 | 0.1231 |
| 5 | 0.7210 | 0.0772 |
| 6 | 0.7400 | 0.0581 |
| 7 | 0.7614 | 0.0368 |
| 8 | 0.7709 | 0.0272 |
| 9 | 0.7982 | - |
| 10 | 0.8062 | 0.0080 |
| 11 | 0.8175 | 0.0193 |
| 12 | 0.8356 | 0.0375 |
| 13 | 0.9294 | 0.1313 |
| 14 | 0.9570 | 0.1588 |
| 15 | 1.1361 | 0.3379 |
| 16 | 1.2159 | 0.4177 |
| 17 | 1.4721 | 0.6739 |

| | |
|-------------|--------|
| Median | 0.7982 |
| N | 17 |
| Sum (X-Med) | 2.6784 |
| AAD | 0.1576 |
| COD | 0.1974 |

$$\text{AAD} = \text{SUM of ABS}(X\text{-Med}) / N$$

$$\text{COD} = \text{AAD} / \text{Median}$$

| N | Ratio |
|----|---------------|
| 1 | 0.5437 |
| 2 | 0.6217 |
| 3 | 0.6577 |
| 4 | 0.6751 |
| 5 | 0.7210 |
| 6 | 0.7400 |
| 7 | 0.7614 |
| 8 | 0.7709 |
| 9 | 0.7982 Median |
| 10 | 0.8062 |
| 11 | 0.8175 |
| 12 | 0.8356 |
| 13 | 0.9294 |
| 14 | 0.9570 |
| 15 | 1.1361 |
| 16 | 1.2159 |
| 17 | 1.4721 |

| | |
|--------|--------|
| Median | 0.7982 |
| COD | 0.1974 |

Sample Size 17 Adjusted COD
for 90% Confidence = .2426

COD of .1974 Compliant

| | | |
|----|--------|--------|
| N | Ratio | |
| 1 | 0.5437 | |
| 2 | 0.6217 | |
| 3 | 0.6577 | |
| 4 | 0.6751 | |
| 5 | 0.7210 | |
| 6 | 0.7400 | LCI |
| 7 | 0.7614 | |
| 8 | 0.7709 | |
| 9 | 0.7982 | Median |
| 10 | 0.8062 | |
| 11 | 0.8175 | |
| 12 | 0.8356 | UCI |
| 13 | 0.9294 | |
| 14 | 0.9570 | |
| 15 | 1.1361 | |
| 16 | 1.2159 | |
| 17 | 1.4721 | |

| | |
|--------------|--------|
| Median | 0.7982 |
| N | 17 |
| COD | 0.1974 |
| SQRT N | 4.1231 |
| 90% CI | 1.6450 |
| Divided by 2 | 3.3913 |
| Plus .05 | 3.4413 |
| Next Integer | 4 |

$$CI = (\text{SQRT } N * 1.645) / 2 + .5$$

| | | | |
|----|--------|--|------------------------------------|
| N | Ratio | | |
| 1 | 0.5437 | | 2018 Net Assessment = \$15,130,987 |
| 2 | 0.6217 | | |
| 3 | 0.6577 | | |
| 4 | 0.6751 | | |
| 5 | 0.7210 | | Plus 5% for statutory compliance |
| 6 | 0.7400 | 2018 Net / LCI = 2019 LCI = \$20,447,280 | \$21,523,452 |
| 7 | 0.7614 | | |
| 8 | 0.7709 | | |
| 9 | 0.7982 | 2018 Net / Median = 2019 100% | \$18,956,817 |
| 10 | 0.8062 | | |
| 11 | 0.8175 | | |
| 12 | 0.8356 | 2018 Net / UCI = 2019 UCI = \$18,107,931 | \$17,245,648 |
| 13 | 0.9294 | | |
| 14 | 0.9570 | | |
| 15 | 1.1361 | | |
| 16 | 1.2159 | | |
| 17 | 1.4721 | | |

| | | | |
|----|--------|--|---|
| N | Ratio | | |
| 1 | 0.5437 | | 2018 Net Assessment = \$15,130,987 |
| 2 | 0.6217 | | |
| 3 | 0.6577 | | |
| 4 | 0.6751 | | |
| 5 | 0.7210 | | Plus 5% for statutory compliance |
| 6 | 0.7400 | 2018 Net / LCI = 2019 LCI = \$20,447,280 | \$21,523,452 |
| 7 | 0.7614 | | |
| 8 | 0.7709 | | |
| 9 | 0.7982 | 2018 Net / Median = 2019 100% | \$18,956,817 |
| 10 | 0.8062 | | |
| 11 | 0.8175 | | |
| 12 | 0.8356 | 2018 Net / UCI = 2019 UCI = \$18,107,931 | \$17,245,648 |
| 13 | 0.9294 | | |
| 14 | 0.9570 | | 2019 Net Assessment = \$15,893,536 |
| 15 | 1.1361 | | |
| 16 | 1.2159 | | |
| 17 | 1.4721 | | |

2019 100%

\$18,956,817

2019 Net Assessment

\$15,893,536

$(2019\ 100\% / 2019\ \text{Net Assessment}) - 1 = \text{order}$

Order = 0.1927

Rounded to 19%



2nd Example COD is Not Compliant

| N | Ratio |
|----|---------------------|
| 1 | 0.2632 |
| 2 | 0.3030 |
| 3 | 0.3500 |
| 4 | 0.4032 |
| 5 | 0.4816 |
| 6 | 0.5097 |
| 7 | 0.5319 |
| 8 | 0.5823 |
| 9 | 0.6962 Median .7548 |
| 10 | 0.8135 Median .7548 |
| 11 | 0.9870 |
| 12 | 1.1361 |
| 13 | 1.1470 |
| 14 | 1.2159 |
| 15 | 1.2638 |
| 16 | 1.4721 |
| 17 | 1.5306 |
| 18 | 1.8526 |

| | |
|--------|--------|
| Median | 0.7548 |
| Mean | 0.8633 |
| N | 18 |

Median = $(N + 1) / 2 =$
 $(18 + 1) / 2 = 9.5$
 observation

Average of the 9th and 10th
 observations

| N | Ratio | X-Med |
|----|--------|--------|
| 1 | 0.2632 | 0.4917 |
| 2 | 0.3030 | 0.4518 |
| 3 | 0.3500 | 0.4048 |
| 4 | 0.4032 | 0.3516 |
| 5 | 0.4816 | 0.2732 |
| 6 | 0.5097 | 0.2451 |
| 7 | 0.5319 | 0.2229 |
| 8 | 0.5823 | 0.1726 |
| 9 | 0.6962 | 0.0587 |
| 10 | 0.8135 | 0.0587 |
| 11 | 0.9870 | 0.2322 |
| 12 | 1.1361 | 0.3812 |
| 13 | 1.1470 | 0.3922 |
| 14 | 1.2159 | 0.4611 |
| 15 | 1.2638 | 0.5089 |
| 16 | 1.4721 | 0.7173 |
| 17 | 1.5306 | 0.7758 |
| 18 | 1.8526 | 1.0977 |

| | |
|-----------------|--------|
| Median | 0.7548 |
| N | 18 |
| Sum ABS (X-Med) | 7.2975 |
| AAD | .4054 |
| COD | .5371 |

| N | Ratio | |
|----|--------|--------------|
| 1 | 0.2632 | |
| 2 | 0.3030 | |
| 3 | 0.3500 | |
| 4 | 0.4032 | |
| 5 | 0.4816 | |
| 6 | 0.5097 | |
| 7 | 0.5319 | |
| 8 | 0.5823 | |
| 9 | 0.6962 | Median .7548 |
| 10 | 0.8135 | Median .7548 |
| 11 | 0.9870 | |
| 12 | 1.1361 | |
| 13 | 1.1470 | |
| 14 | 1.2159 | |
| 15 | 1.2638 | |
| 16 | 1.4721 | |
| 17 | 1.5306 | |
| 18 | 1.8526 | |

| | |
|--------|--------|
| Median | 0.7548 |
| N | 18 |
| COD | 0.5371 |

COD is not compliant with

Sample Size 18 COD of
.2414

| N | Ratio | | |
|----|--------|---------------------------------------|---|
| 1 | 0.2632 | | 2018 Net Assessment \$15,130,987 |
| 2 | 0.3030 | | |
| 3 | 0.3500 | | |
| 4 | 0.4032 | | |
| 5 | 0.4816 | | |
| 6 | 0.5097 | | 20% Above and Below 100% Value |
| 7 | 0.5319 | | \$25,057,941 |
| 8 | 0.5823 | | |
| 9 | 0.6962 | 2018 Net / Median .7548 = 2019 100% = | \$20,046,353 |
| 10 | 0.8135 | | |
| 11 | 0.9870 | | \$16,705,294 |
| 12 | 1.1361 | | |
| 13 | 1.1470 | | |
| 14 | 1.2159 | | |
| 15 | 1.2638 | | 2019 Net Assessment \$15,893,536 |
| 16 | 1.4721 | | Not Compliant |
| 17 | 1.5306 | | |
| 18 | 1.8526 | | |

2019 100%

\$20,046,353

2019 Net Assessment

\$15,893,536

$(2019\ 100\% / 2019\ \text{Net Assessment}) - 1 = \text{order}$

Order = 0.2612

Rounded to 26%

- Ratio Study
- Median
- Mean
- Coefficient of Dispersion
- Sample Size Adjusted Coefficient of Dispersion
- Upper and Lower Confidence Intervals
- Conversion into Aggregate Assessed Values

Summary

- Questions